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Part A, #26

What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at $(3,2)$

implicit differentiation

AB 26

chain

Product + chain

$$\frac{d}{dx}(3y^2) - \frac{d}{dx}(2x^2) = \frac{d}{dx}(6) - \frac{d}{dx}(2xy)$$

$$6y\frac{dy}{dx} - 4x = -2\left(\frac{d}{dx}(x)\,y + x\cdot\frac{d}{dx}(y)\right)$$

$$6y\frac{dy}{dx} - 4x = -2\left(y + x\frac{dy}{dx}\right) = -2y - 2x\frac{dy}{dx}$$

$+2x\frac{dy}{dx}$ $+4x$ $+4x$ $+2x\frac{dy}{dx}$

$$(6y + 2x)\frac{dy}{dx} = 4x - 2y$$

@ (3,2)

$$(12+6)\frac{dy}{dx} = 12 - 4 = 8$$

$$\frac{dy}{dx} = \frac{8}{18} = \frac{4}{9}$$

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Part A #27

Let $f(x)=x^3+x$. If $g(x)=f^{-1}(x)$ and $g(2)=1$, what is the value of $g'(2)$?

Note $g(x)=f^{-1}(x)$

$\Rightarrow f(g(x))=x$

$$\frac{d}{dx}\left(f(g(x))\right)=\frac{d}{dx}(x)$$

$$f'(g(x))\cdot g'(x)=1$$

$$g'(x)=\frac{1}{f'(g(x))}$$

$$f'(x)=\frac{d}{dx}\left(x^3+x\right)=3x^2+1$$

$$f'(g(2))=f'(1)=4$$

so

$$g'(2)=\frac{1}{4}$$

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AB part A #11

Using the substitution $u=2x+1$, $\int_0^2 \sqrt{2x+1}\,dx$

$$\int_{x=0}^{x=2} \sqrt{2x+1}\,dx$$

$u=2x+1$
$du=2\,dx$

So when $x=0$
$u=2(0)+1=1$
when $x=2$,
$u=2(2)+1=5$

oops stop here

$$\frac{1}{2}\int_1^5 \sqrt{u}\,du \quad u^{1/2}$$

$$=\left(\frac{1}{2}\cdot\frac{2}{3}\cdot u^{3/2}\right)\Big|_1^5$$

$$=\frac{1}{3}\sqrt{125}-\frac{1}{3}$$

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part A #21

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$$\int_0^1 e^{-4x}\,dx$$

$u=-4x$

$du=-4\,dx$

$-\frac{1}{4}du = dx$

$x=0 \Rightarrow u=-4(0)$
$=0$

$x=1 \Rightarrow u=-4(1)$
$=-4$

$$-\frac{1}{4}\int_0^{-4} e^u\,du$$

$$= -\left(-\frac{1}{4}\int_{-4}^{0} e^u\,du\right)$$

$$=\frac{1}{4}\left(e^0-e^{-4}\right)=\frac{1}{4}\left(1-\frac{1}{e^4}\right)$$

OR $-\frac{1}{4}\int_0^{-4} e^u\,du =$

$-\frac{1}{4}\left(e^{-4}-e^0\right)$

interleave

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5-6

66

$$\int_0^{\pi/4} \sec^2 x \tan^2 x \, dx$$

$\sec x \tan x (\sec x \tan x \, dx)$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

Let $u = \sec x$

$du = \sec x \tan x \, dx$

$x = 0 \Rightarrow u = 0$

$x = \frac{\pi}{4} \Rightarrow u = \sqrt{2}$

$$\int_0^{\sqrt{2}} u\sqrt{u^2-1} \, du$$

AH HA!

$$\int_0^{\pi/4} \sec^2 x \tan^2 x \, dx$$

$u = \tan x$

$du = \sec^2 x \, dx$

$x = 0 \Rightarrow u = 0$

$x = \pi/4 \Rightarrow u = 1$

$$\int_0^1 u^2 \, du$$

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$$\cos(x+x) = \cos^2 x - \sin^2 x$$
$$= (1-\sin^2 x) - \sin^2 x$$

So

$$\cos(2x) = 1 - 2\sin^2 x$$
$$2\sin^2 x = 1 - \cos(2x)$$
$$\boxed{\sin^2 x = \frac{1-\cos(2x)}{2}}$$

double? half? angle identity

So $$\int \sin^2\left(\theta + \frac{\pi}{6}\right) d\theta$$
$$= \int \frac{1-\cos\left(2\left(\theta+\frac{\pi}{6}\right)\right)}{2}\, d\theta$$

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next worser

AB/A/23 $\frac{d}{dx}\left(\int_0^{x^2} \sin(t^3)\,dt\right)$

FUNDAMENTAL THEOREM OF CALCULUS (weird part)

We know $\frac{d}{dx}\int_0^{x} f(t)\,dt = f(x)$ → but we don't have a plain x, so we need chain rule!

$\frac{d}{dx}(F(x^2) - F(0))$

$f(x^2)\cdot\frac{d}{dx}(x^2)$

$$\frac{d}{dx}\left(\int_0^{x^2} \sin(t^3)\,dt\right) = \sin((x^2)^3)\cdot\frac{d}{dx}(x^2)$$

$$= 2x\sin(x^6)$$

ANSWER

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FTC:

$$\frac{d}{dx}\int_0^x f(t)\,dt = f(x)$$

VALUE of FTC is skip all middle

$$\frac{d}{dx}\int_0^x 3t^2\,dt = \frac{d}{dx}\left((t^3)\Big|_0^x \right)$$

$$= \frac{d}{dx}(x^3 - 0) = 3x^2$$

$f(x) = x^2$

$f(y) = y^2$

$f(c) = c^2$

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last "19%" of part A #19

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A curve has slope $2x+3$ at every (x,y) on the curve. What is the equation of this curve if it goes thru $(1,2)$?

$\frac{dy}{dx} = 2x+3$, so $y = x^2+3x+C$

if $(1,2)$ is on this curve, then

$2 = 1^2+3(1)+C = 4+C$

so $C=-2$

curve is: $y = x^2+3x-2$

$dy = (2x+3)\,dx$

$\int dy = \int 2x+3\,dx$

$y = x^2+3x+C$

substitute "initial value"

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